Zero Assignment for Robust H_2/H_∞ Fault Detection Filter Design

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Abstract

In practical engineering, it is inevitable that a system is perturbed by noise signals. Unfortunately, H_∞/H_∞ filtering may fail to detect some faults when the noise distribution matrix are the same as the fault distribution matrix. In this paper, it is shown that the dynamic feedback gain of a dynamic filter introduces additional zeros to the filter, and both the filter poles and the additional zeros can be assigned arbitrarily. In order to attenuate band-limited noises, the zero assignment technique is used, and an optimal dynamic fault detection filtering approach is proposed by locating the zeros to the noise frequencies and optimizing the poles. Compared to other dynamic filter design approaches, the zero assignment technique gives a better trade-off between more design freedom and computation costs. As shown in the simulation, a better noise attenuation and fault detection performance have been obtained. The zero assignment in multivariable fault detection filter design would be the main contribution of this paper.

Index Terms

Robust Filtering, Zero Assignment, Fault Detection, Fast Fourier Transformation

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I. INTRODUCTION

The fault detection filter problem can be formulated as an estimation problem where system outputs are estimated and certain robustness and sensitivity performance must be satisfied. During the last two decades, the robustness of a fault detection filter has been a central theme in the development of fault detection and isolation (FDI) system (e.g., [1], [2], [3], [4], [5], [6]), and a variety of approaches have been proposed, such as UIOs (Unknown Input Observers) [7], Kalman filters [8], [9], filters with error covariance assignment (ECA) [10], robust filters [11], [12], H_∞ filtering [13] by means of a set of Riccati equations [14], [15] or a set of linear matrix inequations (LMIs) [16] or LMIs with weighting functions [6], [17], H_2/H_∞ filtering [16], [18], [19] and stochastic filtering [20]. The H_2 filter minimises the H_2 norm of the residual transfer function matrix under the assumption that the noises have known power spectral densities. H_∞ filters are good at dealing with deterministic bounded disturbances caused by model uncertainties in order to guarantee some robustness performance. The guaranteed performance, however, may be very conservative, as it is only optimised for the worst-case [6]. Similarly, H_- norm is used to enhance the effects of faults by maximising the minimum (singular) value of the fault transfer function matrix.

It is noted that most of the existing fault detection filters have been simply confined in traditional static filters [2], [15], [16], [21]. Here, the term *static filter* is used to denote the classic Kalman/Luenberger filter, in which a constant gain is used to filter the residual signal. It is well known that the nonunique solution to the gain matrix brings the freedom to design an optimal filter. However, the static filter is able to shift the poles only, but its zeros are invariant. As the performance depends not only on poles, but also on positions of zeros, the zero invariance property imposes a limitation on the performance of disturbance attenuation.

Therefore, it is a natural idea to introduce additional dynamics into filters for modifying the zeros. In order to distinguish from classic filters, the term *dynamic filter* is used, in which a dynamic system is employed to feedback the residual signal. Comparing to the static filters with only one gain matrix, dynamic filters provide more design freedom, and presents both advantages and challenges.

Some preliminary works have been done on dynamic filters, but the attention is mainly on the poles assignment. PIO (Proportional Integral Observer) and PMIO (Proportional Multiple Integral Observer) are discussed in [22], [23]. In [24], a dynamic observer design method is proposed as a dual of control design for the state estimation. A similar work is the Lipschitz UIO [7], where two dynamic compensators are introduced to tackle Lipschitz nonlinearities. It is worth noting that, all the reports on dynamic filter (observer) design ignored the additional zeros introduced by the additional dynamics. The extra free parameters provided by the dynamic filters were determined roughly by optimisation algorithms. From the view point of system performance, the poles are insufficient for achieving an optimal performance. It is felt that the advantages of taking zeros into account would be twofold: 1) it is more possible to attenuate the disturbance further if the filter zeros are close to the disturbance frequency; 2) the specification of zeros puts more constraints on the free parameters and diminishes the search space such that the computation burden is reduced.

Although the multivariable system zeros were first proposed by Rosenbrock over thirty years ago, the system zeros study received relatively less attention compared to the poles research. For more information on system zeros, please see [25], [26], [27]. To the best of our knowledge, there have been no known results of utilizing the zero assignment technique to design a fault detection filter.

Different from all the reported results on dynamic filter design, this paper aims to establish a zero assignment approach for dynamic filter design and do systematic study on its properties. Based on the well-established dynamic state feedback controller design, the properties of filter zeros, the possibility of zeros assignment are studied in section II and III. In section IV, a detailed design procedure is given. An application to fault detection of a multivariable system and its results are illustrated in section V. It has been shown that the zero assignment is possible only in dynamic filters and a better disturbance attenuation performance can be achieved.

II. Problem Formulation

Consider a completely controllable and observable continuous LTI multivariable system

$$\begin{cases} \dot{x}(t) &= \mathbf{A}x(t) + \mathbf{B}u(t) \\ y(t) &= \mathbf{C}x(t) + \mathbf{D}u(t) \end{cases} \tag{1}$$

where $x \in \mathbb{R}^n$ is the state, $u \in \mathbb{R}^p$ the input, $y \in \mathbb{R}^r$ the output ($r \geq p$) and $\mathbf{A} \in \mathbb{R}^{n\times n}$, $\mathbf{B} \in \mathbb{R}^{n\times p}$, $\mathbf{C} \in \mathbb{R}^{r\times n}$, $\mathbf{D} \in \mathbb{R}^{r\times p}$, a faulty system can be presented as [2]:

$$\begin{cases} \dot{x}(t) = \mathbf{A}x(t) + \mathbf{B}u(t) + \mathbf{B}_f f(t) + \mathbf{B}_d d(t) \\ y(t) = \mathbf{C}x(t) + \mathbf{D}u(t) + \mathbf{D}_f f(t) + \mathbf{D}_d d(t) \end{cases} \tag{2}$$

where $f(t) \in \mathbb{R}^f$, $d(t) \in \mathbb{R}^d$ are the general fault vector and disturbance vector, respectively. $B_d(B_f)$, $D_d(D_f)$ are disturbance(fault) distribution matrices. $d(t)$ is a general disturbance vector due to exogenous signals, linearisation or parameter uncertainties. For instance, model uncertainties can be presented as:

$$d(t) = \Delta A x(t) + \Delta B u(t) \tag{3}$$

Without loss of generality, $d(t)$ is assumed as a quasi-stationary process with both deterministic and stochastic components:

$$d(t) = s(t) + h(t) * n(t) \tag{4}$$

where $s(t)$ is a deterministic bounded disturbance with band-limited spectrum, $n(t)$ a white noise, $h(t)$ the impulse response of a band-pass filter having the similar band as $s(t)$ and $*$ denotes the convolution product. Thus, $h(t) * n(t)$ is a band-limited stationary stochastic signal (colored noise) and $d(t)$ is quasi-stationary and band-limited.

The fault distribution matrices B_f, D_f can be determined according to which faults are to be detected. For sensor faults, they are

$$\begin{cases} B_f = 0 \\ D_f = I_r \end{cases} \tag{5}$$

For actuator faults, they are

$$\begin{cases} B_f = B \\ D_f = D \end{cases} \tag{6}$$

For both the disturbance/fault free system (1) and corrupted system (2), a mth-order dynamic filter will be used throughout this paper with:

$$\begin{cases} \dot{z}(t) = \mathbf{K}_1 z(t) + \mathbf{K}_2 r(t) \\ v(t) = \mathbf{K}_3 z(t) + \mathbf{K}_4 r(t) \end{cases} \tag{7}$$

and

$$\begin{cases} \dot{\hat{x}}(t) = \mathbf{A}\hat{x}(t) + \mathbf{B}u(t) + v(t) \\ \hat{y}(t) = \mathbf{C}\hat{x}(t) + \mathbf{D}u(t) \end{cases} \tag{8}$$

where $z \in \mathbb{R}^m$ is the *dynamic feedback state vector*, $v \in \mathbb{R}^n$ the output of dynamic feedback, and

$$r(t) = y(t) - \hat{y}(t) \tag{9}$$

is the residual signal. The basic concept of robust fault detection filter is to detect $f(t)$ from the residual signal. $r(t)$ also works as a correction term to reduce the negative effects due to $d(t)$.

The block diagram of the filtering problem is depicted in Fig.1. This dynamic filter has the similar forward model (8) as the classic static filter. The obvious difference between the static filter and the dynamic filter is the feedback path: the real coefficient constant gain matrix K in the static filter is replaced with a dynamic system (7). This new dynamic system (7) offers more freedom which will be used to assign zeros. The *transfer function matrix* (TFM) of the dynamic feedback system (7) relating r to v is given by

$$H(s) = \mathbf{K}_3(sI - \mathbf{K}_1)^{-1}\mathbf{K}_2 + \mathbf{K}_4 \tag{10}$$

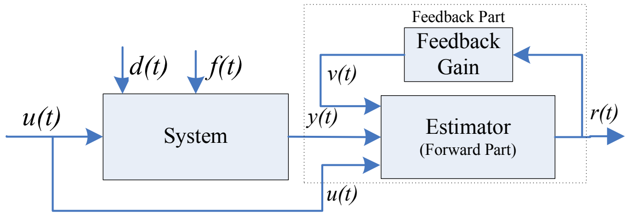

Fig. 1. Structure of the filter problem

Note that, in static filters, the feedback gain is a real coefficient constant matrix K without frequency complex variable s, which leads its TFM is a constant matrix K. Hence, the static filter does not change the frequency characteristics of the correction term $r(t)$. In dynamic filters, however, the feedback part is a dynamic system (7)with TFM (10) with more freedom. Furthermore, it can been seen that a static filter is a particular case of dynamic filters when setting $K_1 = 0, K_2 = 0$ and $K_3 = 0$.

By connecting the dynamic feedback (7) and the forward part (8), the overall dynamics of the dynamic filter can be rewritten in an augment form:

$$\begin{cases} \begin{pmatrix} \dot{\hat{x}} \\ \dot{z} \end{pmatrix} = \begin{bmatrix} \mathbf{A} - \mathbf{K_4C} & \mathbf{K}_3 \\ -\mathbf{K}_2\mathbf{C} & \mathbf{K}_1 \end{bmatrix} \begin{pmatrix} \hat{x} \\ z \end{pmatrix} + \begin{bmatrix} \mathbf{B} \\ \mathbf{0} \end{bmatrix} u \\ \qquad + \begin{bmatrix} \mathbf{K}_4 \\ \mathbf{K}_2 \end{bmatrix} (y - \mathbf{D}u) \\ \hat{y} = [\mathbf{C} \quad \mathbf{0}] \begin{pmatrix} \hat{x} \\ z \end{pmatrix} + \mathbf{D}u \end{cases} \tag{11}$$

Defining $e(t) = x(t) - \hat{x}(t)$ and subtracting (8) from the disturbance/fault corrupted system (2) yield

$$\begin{cases} \begin{pmatrix} \dot{e}(t) \\ \dot{z}(t) \end{pmatrix} = \begin{bmatrix} \mathbf{A} - \mathbf{K_4C} & -\mathbf{K}_3 \\ \mathbf{K}_2\mathbf{C} & \mathbf{K}_1 \end{bmatrix} \begin{pmatrix} e(t) \\ z(t) \end{pmatrix} \\ \qquad + \begin{bmatrix} \mathbf{B}_d - \mathbf{K}_4\mathbf{D}_d \\ \mathbf{K}_2\mathbf{D}_d \end{bmatrix} d(t) + \begin{bmatrix} \mathbf{B}_f - \mathbf{K}_4\mathbf{D}_f \\ \mathbf{K}_2\mathbf{D}_f \end{bmatrix} f(t) \\ r(t) = [\mathbf{C} \quad 0] \begin{pmatrix} e(t) \\ z(t) \end{pmatrix} + \mathbf{D}_d d(t) + \mathbf{D}_f f(t) \end{cases} \tag{12}$$

It can be seen that the residual signal $r(t)$ depends both on the disturbance $d(t)$ and the fault $f(t)$. Due to the existence of $d(t)$, the fault detection performance may be considerable degraded. It is essential to attenuate the effects of $d(t)$ in $r(t)$ and enhance the sensitivity of $r(t)$ to $f(t)$.

Equation (12) can be rewritten in a compact form:

$$\begin{cases} \dot{\widetilde{x}}(t) & = \widetilde{A}\widetilde{x}(t) + \widetilde{B}_d d(t) + \widetilde{B}_f f(t) \\ r(t) & = \widetilde{C}\widetilde{x}(t) + \widetilde{D}_d d(t) + \widetilde{D}_f f(t) \end{cases} \tag{13}$$

where

$$\widetilde{x} = [e^T(t)\ z^T(t)]^T, \qquad \widetilde{A} = \begin{bmatrix} \mathbf{A} - \mathbf{K}_4\mathbf{C} & -\mathbf{K}_3 \\ \mathbf{K}_2\mathbf{C} & \mathbf{K}_1 \end{bmatrix},$$

$$\widetilde{B}_d = \begin{bmatrix} \mathbf{B}_d - \mathbf{K}_4\mathbf{D}_d \\ \mathbf{K}_2\mathbf{D}_d \end{bmatrix}, \quad \widetilde{B}_f = \begin{bmatrix} \mathbf{B}_f - \mathbf{K}_4\mathbf{D}_f \\ \mathbf{K}_2\mathbf{D}_f \end{bmatrix},$$

$$\widetilde{C} = \begin{bmatrix} \mathbf{C} & 0 \end{bmatrix}, \qquad \widetilde{D}_d = \mathbf{D}_d, \quad \widetilde{D}_f = \mathbf{D}_f$$

There are various methods to design the dynamic filter parameters K_1, K_2, K_3, K_4, such as eigen-structure assignment [21], and dual controller design approach [24], [7]. In this paper, in order to get the desired fault detection performance, we study the properties of the dynamic filter zeros first, and propose an approach utilizing the zero assignment methodology.

III. TFMs of Dynamic Filter

It can be seen from (13) that $r(t)$ is not affected by the system input $u(t)$, as the process dynamics are canceled in the observer. However, both $f(t)$ and $d(t)$ contribute to non-zero $r(t)$. Assuming the initial conditions are zero, s-transforming (13) gives the TFM relating $d(t)$ to $r(t)$

$$\widetilde{G}_d(s) \;\; = \;\; \widetilde{C}(sI - \widetilde{A})^{-1}\widetilde{B}_d + \widetilde{D}_d\ . \tag{14}$$

Similarly, the TFM relating $f(t)$ to $r(t)$ is given by

$$\widetilde{G}_f(s) \;\; = \;\; \widetilde{C}(sI - \widetilde{A})^{-1}\widetilde{B}_f + \widetilde{D}_f \tag{15}$$

The whole dynamics of the dynamic filter can be expressed as:

$$r(s) = \widetilde{G}_f(s)f(s) + \widetilde{G}_d(s)d(s) \tag{16}$$

It can be seen clearly from (16) that, due to the existence of $d(s)$, the residual $r(t)$ is nonzero even when there is no fault. For a successful fault detection, it is essential to make $\widetilde{G}_d(s)$ small and enlarge $\widetilde{G}_f(s)$.

A. Poles of Dynamic Filters

From the simplified expression (13) of the dynamic filter and its TFMs, one can see that the stability of the dynamic filter is determined by the matrix $\widetilde{A}$. According to linear system theory, the poles of the filter (13) are the roots of the characteristic polynomial $\det(sI - \tilde{A}) = 0$. It follows that the complete set of poles coincides with the eigenvalues of the matrix $\widetilde{A}$. The dynamic filter is stable if and only if all the eigenvalues of $\widetilde{A}$ are in the left half s-plane.

It is worthy noting that, because of the importance of poles, most filter design approaches (e.g., eigenstructure assignment) in literature concerned on the positions of poles and ignored the zeros. We will analyze the zeros of the dynamic filter and assign the zeros by placing appropriate values to some free parameters.

B. Zeros of Dynamic Filter

During the last three decades, considerable research has been done on defining zeros (called transmission zeros or invariant zeros) and deriving their properties. Generally, the transmission zeros are defined in terms of TFM [28]. The transmission zeros are the complex numbers such that the rank of the TFM is locally reduced. It has been shown that, if s is a zero, then there exists some non zero proportional e^{st} input vector such that its propagating through the system is blocked [25]. In this paper, this property will be used to attenuate the propagation of $d(s)$ in (16). First of all, the disturbance zeros are defined in an analogous way as the definition of transmission zeros.

Definition 1 (*disturbance zeros of the actual system*): the disturbance zeros of the plant (2) are these transmission zeros of the TFM relating the disturbance $d(t)$ to the system output $y(t)$. That is the set of complex number s such that $G_d(s)$ loses rank locally

$$Z_1 = \{s|\ \text{rank}\ G_d(s) < \min(r, d)\} \tag{17}$$

where r, d are the dimension of the residual and disturbance, respectively, and $G_d(s) = C(sI - A)^{-1}B_d + D_d$.

Note that these disturbance zeros defined in (17) are different from the system (input/output) transmission zeros. The disturbance zeros $\{Z_1\}$ are related to the TFM from $d(t)$ to $y(t)$, whereas the transmission zeros are associated with TFM $G(s) = C(sI - A)^{-1}B + D$ relating $u(t)$ to $y(t)$.

Definition 2 (*disturbance zeros of the dynamic filter*): the disturbance zeros of the filter (13) are the transmission zeros of $\widetilde{G}_d(s)$ such that $\widetilde{G}_d(s)$ loses rank locally.

$$Z_2 = \{s|\ \text{rank}\ \widetilde{G}_d(s) < \min(r, d)\} \tag{18}$$

It is worthy noting that disturbance zeros Z_2 may vary from Z_1, as $\widetilde{G}_d(s)$ differs from $G_d(s)$. The relationship between these two sets will be given in the following theorem. Before presenting the theorem, an existing lemma shows that zeros are associated with reducing a column or row rank of the (Rosenbrock) system matrix.

Lemma [28], [26] Given a completely controllable and observable system (A, B, C, D), for any complex number s, the rank of the (Rosenbrock) system matrix $P(s)$ is equal to the rank of the TFM $F(s) = C(sI - A)^{-1}B + D$ plus n.

$$\text{rank } P(s) = \text{rank } F(s) + n$$

where

$$P(s) = \begin{bmatrix} sI - A & B \\ -C & D \end{bmatrix}$$

This lemma can be understood as that $F(s)$ loses rank at a complex frequency $s = z$ if and only if the normal rank of the system matrix $P(s)$ is reduced at $s = z$. A useful conclusion from the lemma is that, under the condition of controllability and observability, the zero set is the same as the set of complex numbers $\{s\}$ at which the system matrix $P(s)$ loses rank locally.

Theorem 3.1: If the system (A, B_d, C, D_d) is controllable and observable, the disturbance zeros Z_2 of the dynamic filter (13) are the disturbance zeros Z_1 of the plant system (2) together with the poles of the dynamic filter gain (7, namely

$$Z_2 = Z_1 \cup \{\text{eigenvalues of } K_1\} \tag{19}$$

Proof: According to the lemma, the disturbance zeros of $G_d(s)$ in system (2) coincides with those complex values s at which the Rosenbrock system matrix $P_d(s)$ loses rank locally.

$$Z_1 = \{s|\text{rank}P_d(s) < n + \min(r, d)\} \tag{20}$$

where

$$P_d(s) = \begin{bmatrix} sI - A & B_d \\ -C & D_d \end{bmatrix} \tag{21}$$

with normal rank $n + \min(r, d)$.

Similarly, for the dynamic filter (13), the disturbance zeros are the set of complex numbers s such that

$$\text{rank } \widetilde{P}_d(s) < n + m + \min(r, d) \tag{22}$$

where

$$\widetilde{P}_d(s) = \begin{bmatrix} sI - A + K_4C & K_3 & B_d - K_4D_d \\ -K_2C & sI - K_1 & K_2D_d \\ -C & 0 & D_d \end{bmatrix} \tag{23}$$

is the corresponding Rosenbrock system matrix with size $(n+m+r) \times (n+m+d)$ and normal rank $n + m + \min(r, d)$. Hence, the disturbance zeros of $\widetilde{G}_d(s)$ can be expressed as

$$Z_2 = \{s|\text{rank } \widetilde{P}_d(s) < n + m + \text{min}(r, d)\} \tag{24}$$

The rank of $\widetilde{P}_d(s)$ (23) can be calculated by

$$\begin{aligned} & \text{rank } \widetilde{P}_d(s) \\ = & \text{ rank} \begin{bmatrix} sI - A & K_3 & B_d \\ -K_2C & sI - K_1 & K_2D_d \\ -C & 0 & D_d \end{bmatrix} \\ = & \text{ rank} \begin{bmatrix} sI - A & K_3 & B_d \\ 0 & sI - K_1 & 0 \\ -C & 0 & D_d \end{bmatrix} \\ = & \text{ rank} \begin{bmatrix} sI - A & B_d & K_3 \\ -C & D_d & 0 \\ 0 & 0 & sI - K_1 \end{bmatrix} \end{aligned} \tag{25}$$

It follows that the filter disturbance zeros Z_2 are those values of s for which

$$\text{rank} \begin{bmatrix} sI - A & B_d \\ -C & D_d \end{bmatrix} < n + \text{min}(r, d) \tag{26}$$

or/and

$$\text{rank } [sI - K_1] < m \tag{27}$$

Comparing (26) and (20), one can see that the zeros given by equation (26) coincide with Z_1. Then the disturbance zeros of the plant system is a subset of the disturbance zeros of the corresponding dynamic filter.

Equation (27) shows that the eigenvalues of K_1 compose another subset of the dynamic filter disturbance zeros. These zeros are the poles of the dynamic gain (7).

The result of the theorem follows. **Q.E.D.**

Theorem 3.1 verifies that the disturbance zeros are invariant in static filter. It means, if s is a disturbance zero of plant (2), s is a zero of the corresponding dynamic/static filter too. Due to the zero invariance, one can not shift the positions of zeros in the static filter. In dynamic filters, however, the extra disturbance zeros introduced by the dynamic gain can be arbitrarily assigned, even if some of the filter disturbance zeros are invariant from the system disturbance zeros. This is the main implication of Theorem 3.1.

Theorem 3.1 implies that a mth-order dynamic system (7) introduces m additional transmission zeros in the dynamic filter, and the additional zeros are located at the poles of the dynamic feedback gain. Theorem 3.1 can be understood as a generalization of the well-known SISO dynamic feedback control result that closed-loop zeros are zeros in the forward-path and poles in the feedback-path.

Remark 1: From Theorem 3.1 it follows that PI filters proposed by [22] only introduces disturbance zeros at the origin. This explains why PI filters achieve a better performance at rejecting step/constant disturbances in the steady state than static filters.

Remark 2: It is of interest to draw attention to the dynamic filter in [7], where two dynamic systems are adopted to replace two constant gain matrices. It can be seen that, by adding two dynamic feedbacks (one added into $\dot{\hat{x}}$, one into $\hat{y}$), more parameters and design freedom are provided than that of the present dynamic filter. It is of interest and encouraged to extend the present work by the use of this kind of filters in the future.

Remark 3: The proposed dynamic filter is a robust filter, which differs from the Kalman Filter (KF) in two ways: 1) different structures, the KF has the same structure as the static filter, and the dynamic filter extends the static gain matrix to a dynamic system; 2) different performance criteria. The KF aims at minimizing the covariance of state estimation error under stochastic noises. The objective of the robust filter is to enhance robustness to model uncertainties and/or deterministic disturbances. Therefore, the resulting filters show different performances. As reported in many literatures [15], the KF works well in rejecting stochastic noises with known covariance, and robust filters achieve a better performance in attenuating deterministic disturbances. Furthermore, the advantages of the dynamic filter are: a) more design freedom; b) by assigning zeroes, the number of free parameters is reduced by $m \times m$.

Without claiming neither that Kalman filtering is not useful in fault detection, nor that the (dynamic/static) filter is the best one available, it is felt that, as a different structure, the robust dynamic filter is an alternative which can give a new sight on filter design and achieve a better robust performance.

IV. ZERO-POLE ASSIGNMENT PROCEDURE

It is well known that poles are insufficient to determine the system behavior that is also greatly affected by zeros. For instance, in a SISO system, if the transfer function has a zero at $\pm j\omega$, then the magnitude response at frequency w is zero. Thus, the steady-state system output of the sinusoidal input $\alpha sin(wt)$ is zero. According to the theory of system zero [28], [25], if s is the zero of a MIMO system, then there exists some input (vector) $u(t) = \beta e^{st}, (\beta \neq 0)$ such that its propagating through the system is blocked. Since the disturbance zeros Z_2 are defined as the zeros between $d(t)$ and $r(t)$, for some specific disturbance, $d(t)$ can not pass through the filter. One can use this property to obtain disturbance attenuation. Hence, placing one or more disturbance zeros at the origin (or $\pm j\omega$ on the imaginary axis) will attenuate a step disturbance (or a sinusoidal disturbance $e^{j\omega t}$, respectively).

Unfortunately, because of the invariance of zeros, the disturbance zeros in static filters can not be changed. Theorem 3.1, however, shows the possibility of zero assignment in dynamic filters by introducing additional zeros. It is also shown that the additional zeros are independent from $K_2,\ K_3$ and K_4. The zero assignment in dynamic filters can be stated as: the additional zeros introduced by K_1 can be arbitrarily placed to desired positions by assigning the eigenvalues of K_1 properly. Thus, the limitation of invariant zeros in static filters is overcome and the disturbance attenuation performance can be improved to some extent.

A further consideration is that the set of zeros must be self-conjugated, such that the resulting matrix K_1 is real. For a disturbance at frequency ω_i, the desired zeros should be $\pm j\omega_i$. Thus, the required number of poles is twice the number of disturbance frequencies and the order of the dynamic feedback (7) can be determined by

$$m = 2n_d \tag{28}$$

where n_d is the number of disturbance frequencies. Hence, the sizes of the parameter matrices $K_1,\ K_2,\ K_3,\ K_4$ are determined accordingly. For pole assignment, the matrix $\widetilde{A}$ can be decomposed as

$$\begin{aligned} \widetilde{A} &= \begin{bmatrix} \mathbf{A}-\mathbf{K}_4\mathbf{C} & -\mathbf{K}_3 \\ \mathbf{K}_2\mathbf{C} & \mathbf{K}_1 \end{bmatrix} \\ &= \begin{bmatrix} \mathbf{A} & 0 \\ 0 & \mathbf{K}_1 \end{bmatrix} + \begin{bmatrix} -\mathbf{K}_4 & -\mathbf{K}_3 \\ \mathbf{K}_2 & 0 \end{bmatrix} \begin{bmatrix} \mathbf{C} & 0 \\ 0 & \mathbf{I} \end{bmatrix} \end{aligned} \tag{29}$$

where A, C is known and K_1 is determined by the zero assignment. The pole assignment is to find an appropriate parameter matrix $\begin{bmatrix} -\mathbf{K}_4 & -\mathbf{K}_3 \\ \mathbf{K}_2 & 0 \end{bmatrix}$ so that the eigenvalues of $\widetilde{A}$ are assigned to some areas on the left-half s-plane.

Remark 4: Note that, as the magnitude response in the neighborhood of the pole shoots up to infinity, the pole regions should be far from the disturbance frequency $\pm j\omega$ to avoid their effects of enlarging the magnitude response at $\pm j\omega$. Similarly, because the major frequency components of abrupt/incipient faults are at low frequency [2], the poles of the fault TFM $\widetilde{G}_f(s)$ should be close to the origin, such that the magnitude response at low frequency is enlarged and the effects of $f(t)$ in residual $r(t)$ can be enhanced.

Remark 5: The benefit of zero assignment is that a better trade-off is achieved between the design freedom and computation complexity. This would be twofold: 1) making use of the design freedom provided by dynamic filters to improve the disturbance attenuation performance; 2) reducing the computation complexity of dynamic filter design. Particularly, it is true for the issue of 'curse of dimensions' in parameter optimization. Compared to static filters where only one $n \times r$ matrix need to be optimized, the number of free parameters in dynamic filters are $(m+n) \times (m+r)$. For most current dynamic filter design approaches, these free parameters are selected roughly by optimization algorithms [22], [24]. The possibility of being trapped in local minima increases as the dimension of parameter increases. With the aid of zero assignment by specifying the matrix K_1, the dimension of the search space in parameter optimization is reduced by $m \times m$.

In summary, the zero assignment technique in dynamic filter design, on one side, is able to attenuate the disturbance further by assigning zeros close to the disturbance frequency, and, on the other side, to reduce the computation complexity by diminishing the search space.

The zero assignment solution to the dynamic robust fault detection filter design can now be stated as follows:

Given a system (2) corrupted by $d(t)$, if the main frequency contents of residuals $r(t)$ can be estimated at $w_i, (i = 1, 2, \ldots, n_d)$, then, **(a)** *assign the zeros of dynamic filter to $\pm jw_i, (i = 1, 2, \ldots, n_d)$;* **(b)** *place the eigenvalues of $\widetilde{A}$ in the left half s-plane and* **(c)** *minimize the following performance index*

$$J = \frac{\sum_i^{n_d} W_i \left|\left|\widetilde{G}_d(s)\right|\right|_{2,s=jw_i}}{\rho + \left|\left|\widetilde{G}_f(s)\right|\right|_\infty} \tag{30}$$

to give the optimal gain matrix K_2, K_3 and K_4. Therefore, residual $r(t)$ is an optimal detection of $f(t)$.

Here, W_i is the weighting factor selected according to the distribution of the disturbance, $\|\cdot\|_{2,s=jw_i}$ denotes the 2-norm of a TFM at $s = jw_i$, $\|\cdot\|_\infty$ denotes the H_∞ norm of a TFM and ρ is a small positive real number to guarantee the denominator will not be zero. The minimization of numerator $\sum_i^{n_d} W_i||\tilde{G}_d||$ is for attenuating disturbances (in another words, to enhance the robustness to disturbances);

the minimization of $1/(\rho + ||\tilde{G}_f||_\infty)$ (equivalently, maximization of $||\tilde{G}_f||_\infty$) is to enhance the effects of faults in residuals $r(t)$. The detailed design procedure is

1) Estimate the disturbance frequencies $\{w_i\}$ via the spectrum analysis of the residual. The residual can be generated by any stable static filter K_0 (e.g., by using MATLAB $place$ command);
2) Determine the order m of the dynamic gain by (28), and assign the eigenvalues of K_1 close to these disturbance frequencies $\{w_i\}$.
3) Select regions of dynamic filter poles.
4) Select initial values of K_2, K_3, K_4 and find their optimal values through optimization such that the performance function J (30) is minimized;
5) A fault can be detected during system operation by using threshold techniques or detecting changes in statistical properties of residuals, e.g., mean values.

The optimization step aims at (a) stabilizing the dynamic filter; (b) enhancing the robustness to disturbances and (c) optimizing the sensitivity to faults. This is a typical constrained optimization problem and can be done by setting nonlinear constraints on the K_2, K_3 and K_4.

V. Application and Results

To illustrate the proposed fault detection filter design approach, this section considers the robust fault detection of a gas turbine engine whose model is given as:

$$\begin{cases} \dot{x}(t) = \begin{bmatrix} -0.943 & 0.1601 \\ 3.9439 & -3.234 \end{bmatrix} x(t) \\ \qquad\qquad + \begin{bmatrix} 86.794 & 40.312 \\ 154.691 & 81.275 \end{bmatrix} u(t) \\ y = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} x(t) \end{cases} \tag{31}$$

The disturbance model is assumed as

$$B_d = B_f = B \qquad D_d = D_f = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \tag{32}$$

where the fault matrix $B_f = B$ for actuator faults. Note that, $B_d = B$ too, as it is common in the industrial applications where disturbances enter the system by corrupting the input signal. A further advantage is that the estimation of disturbance matrix B_d is avoided. It can be verified that the system (31) and (A, B_d, C, D_d) is controllable and observable. Then the condition of Theorem 3.1 is met.

It is worth noting that, in this configuration ($B_d = B_f = B$), the widely used H_∞/H_∞ static filter design may fail without weighting functions, as the TFMs $\widetilde{G}_f(s)$, $\widetilde{G}_d(s)$ are the same and the performance index $\|\widetilde{G}_d(s)\|_\infty/\|\widetilde{G}_f(s)\|_\infty$ is always equal to 1.

The disturbances injected to the systems are quasi-stationary signals:

$$\mathbf{d}(t) = \begin{pmatrix} 0.25[\sin(5t) + \sin(10t + \pi/4)] + n_1(t) \\ 0.25[\cos(5t) + \sin(10t + \pi/3)] + n_2(t) \end{pmatrix} \tag{33}$$

where $n_1(t)$ and $n_2(t)$ are mutually independent white noises $N(0,\ 0.01)$ with zero mean and variance 0.01. Fig. 2 shows the disturbances. In the simulation, the inputs $u(t)$ are unit step signals and both the amplitudes of $d_1(t), d_2(t)$ are over 50% of the amplitudes of input signals. Moreover, the disturbances contain mainly low frequency components. Attenuating low frequency disturbances is given higher priority, because common step/incipient faults are mainly at low frequencies and most of industrial systems behave as low-pass filters. Hence, it is more challenging to attenuate low frequency disturbances.

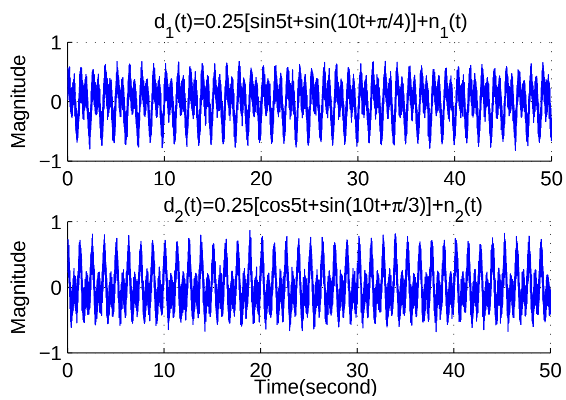

Fig. 2. Quasi-stationary disturbances

Following the procedure in the preceding section, now a dynamic filter is designed for this system.

Step 1. In order to estimate the disturbance frequency, a static filter is first constructed via $K_0 = place(A^T, C^T, [-1, -1])^T$. An 1024-point FFT is employed to calculate the spectrum of its residual $r(t)$. Fig.3 provides an illustration of the spectrum of the actual disturbance and its estimate via the residual. From the plot on the right of Fig.3, it can be seen that $r(t)$ has two main components corresponding to the disturbance frequencies $\omega = \{5.0, 10\}$ (rad/sec). This estimated frequencies agree with the actual disturbance frequencies. Hence, set $n_d = 2$, $\omega_i = \{5.0, 10\}$ and the weighting factors $W_i = 1, i = 1, 2$

Step 2. It follows that the desired zero positions are $\pm 5j,\ \pm 10j$ and the order of the dynamic gain

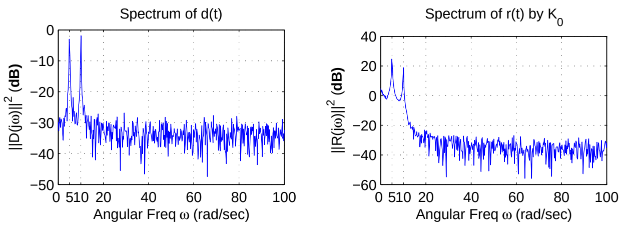

Fig. 3. Spectra of the disturbances and the residuals of K_0.

$$\widetilde{G}_f(s) = \widetilde{G}_d(s) = \frac{\begin{bmatrix} H_{11}(s) & H_{12}(s) \\ H_{21}(s) & H_{22}(s) \end{bmatrix}}{(s^2+1.833s+0.8447)(s^2+2.344s+1.382)} \tag{35}$$

where $H_{11}(s) = 86.79(s+14.75)(s^2+3s+18.2)(s^2+11.3s+75.3)$ $H_{12}(s) = 154.7(s+16.63)(s^2+2.41s+6.62)(s^2-0.713s+35.74)$,
$H_{21}(s) = 40.31(s+14.4)(s^2+2.27s+17.65)(s^2+13.6s+87.26)$ $H_{22}(s) = 81.28(s+16.65)(s^2+2.225s+6.615)(s^2-0.42s+34.78)$.

is $m = 4$. Then the dynamic filter structure is $K_1 \in \mathbb{R}^{4\times4}$, $K_2 \in \mathbb{R}^{4\times2}$, $K_3 \in \mathbb{R}^{2\times4}$, $K_4 \in \mathbb{R}^{2\times2}$. Let

$$K_1 = \begin{bmatrix} 0 & -5 & 0 & 0 \\ 5 & 0 & 0 & 0 \\ 0 & 0 & 0 & -10 \\ 0 & 0 & 10 & 0 \end{bmatrix} \tag{34}$$

to assign zeros to $\{\pm 5j, \pm 10j\}$.

step 3. Set the desired regions of poles are: (a) their real parts are less than -1, and (b) their imaginary parts are close to real axis as much as possible. The reason for (b) is that, if the pole is on real axis without imaginary part, its effects on imaginary frequencies may be reduced. An equivalent statement was presented in *Remark 7*.

step 4. Set the initial values of K_2, K_3, K_4 randomly and use the algorithm *fmincon()* (by Optimization

Toolbox in Matlab). The optimal solution is

$$K_2 = \begin{bmatrix} 0.0727 & 3.7092 \\ 2.5856 & -5.3730 \\ -0.2838 & 5.2500 \\ -6.2984 & 0.0765 \end{bmatrix},$$
$$K_3 = \begin{bmatrix} 1.7224 & -4.5248 & -27.1796 & 6.0431 \\ -3.5649 & -12.4087 & -7.2836 & -8.8425 \end{bmatrix}, \tag{36}$$
$$K_4 = \begin{bmatrix} 18.5227 & -5.0874 \\ 5.9855 & 16.4604 \end{bmatrix}$$

which gives the resulting dynamic filter

$$\widetilde{A} = \begin{bmatrix} -19.47 & 5.248 & -1.72 & 4.523 & 27.18 & -6.04 \\ -2.042 & -19.7 & 3.565 & 12.41 & 7.284 & 8.843 \\ 0.072 & 3.709 & 0 & -5 & 0 & 0 \\ 2.586 & -5.373 & 5 & 0 & 0 & 0 \\ -0.284 & 5.250 & 0 & 0 & 0 & -10 \\ -6.298 & 0.077 & 0 & 0 & 10 & 0 \end{bmatrix}$$
$$\widetilde{B}_d = \widetilde{B}_f = \begin{bmatrix} 86.794 & 40.312 \\ 154.691 & 81.275 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \tag{37}$$
$$\widetilde{C} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix}, \ \widetilde{D}_d = 0, \ \widetilde{D}_f = 0$$

And the six poles of the dynamic filter are:

$$\{-16.8934, \ -15.8096, \ -3.4574 \ -1.0 \pm 0.2617j, \ -1.0\} \tag{38}$$

As discussed before, since the disturbance(fault) distribution matrices $B_d = B_f = B$, the TFMs $\widetilde{G}_f(s)$, $\widetilde{G}_d(s)$ are identical, as shown in equation (35).

It is easy to verify that the normal rank of TFM (35) is 2 and it reduces to 1 when $s = \{\pm 5j, \pm 10j\}$. Therefore, the disturbance zeros of TFM (35) are $\{\pm 5j, \pm 10j\}$ which is consistent with Theorem 3.1.

For fair comparison to the conventional static filter, a H_∞/H_∞ static filter L_1 is designed by taking use of weighting functions. Since the zeros are of interest, the effects of different poles should be reduced as much as possible. Hence, the pole regions are restricted to the similar area as the dynamic filter. The resulting H_∞/H_∞ filter gain matrix is

$$L_1 = \begin{pmatrix} 4.0827 & -3.8591 \\ 7.9752 & -6.2600 \end{pmatrix} \tag{39}$$

which gives the filter poles at $-1.0 \pm 0.0082j$.

A static filter is also designed by using *place()* function provided by Matlab which gives the static gain matrix

$$L_2 = \begin{pmatrix} 0.0574 & -0.1016 \\ 4.2056 & -2.2347 \end{pmatrix} \tag{40}$$

with poles at $-1.0 \pm 0.2617j$. Note that, because the static filters do not change the system order, they have only two poles. These two poles $\{-1.0 \pm 0.2617j\}$ are selected as the desired poles for *place()*, because they are dominant poles in the dynamic filter.

The magnitude responses of $\widetilde{G}_f(s)$ and $\widetilde{G}_d(s)$ of these 3 filters are depicted in Fig. 4. It can be seen that,

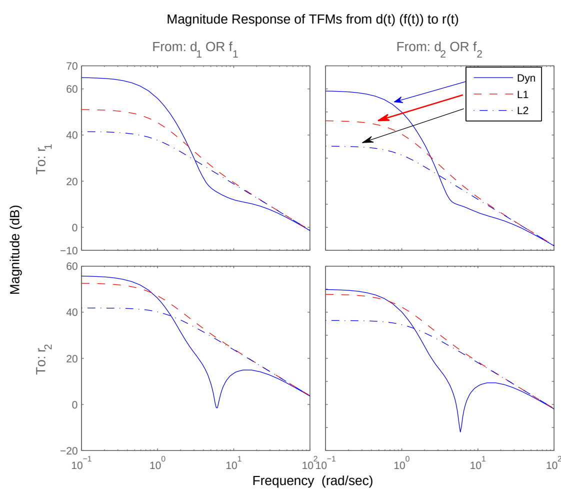

Fig. 4. Magnitude responses of the TFMs $\widetilde{G}_f(s)$ and $\widetilde{G}_d(s)$ in dynamic filter (solid), static filter L_1 (dashed) and L_2 (dash-dot). Note that, $\widetilde{G}_d(s)$ and $\widetilde{G}_f(s)$ are identical, because $B_d = B_f, D_d = D_f$.

compared to those of static filters, the dynamic filter has dips around the disturbance frequencies in the magnitude response. These dips contribute the improvement on attenuating the band-limited disturbances. Furthermore, in the dynamic filter, the magnitude around zero frequency (where the main components of the fault are) is greater than that of static filters, which enhances the effects of the fault in residuals.

A. Residuals without fault

In this simulation, no fault happens. Fig. 5 shows the norms of $r(t)_{dyn}$, $r(t)_{L1}$ and $r(t)_{L2}$, which are the residuals of our dynamic filter $\{K_1, K_2, K_3, K_4\}$, the static filter L_1 and L_2, respectively. It can be seen that $\|r(t)\|_{dyn}$ has a large overshoot at the beginning due to the transient process of the dynamic filter. In the steady state, the disturbance attenuation in dynamic filter is more significant than that of L_1, L_2. In the steady state, the maximum magnitude of $\|r(t)\|_{dyn}$ is below than 8, however that of $\|r(t)\|_{L1}$, $\|r(t)\|_{L2}$ is nearly 20. A comparison has also been made between the proposed method and the classic disturbance estimation method [2] (denoted by L_3) as shown in Figure 5, where the 4th subplot $||r(t)||_{L3}$ shows the residual response of the disturbance estimation method. It can be seen that their maximum magnitude are similar. However, our method gives a smaller standard deviation which is defined as follows.

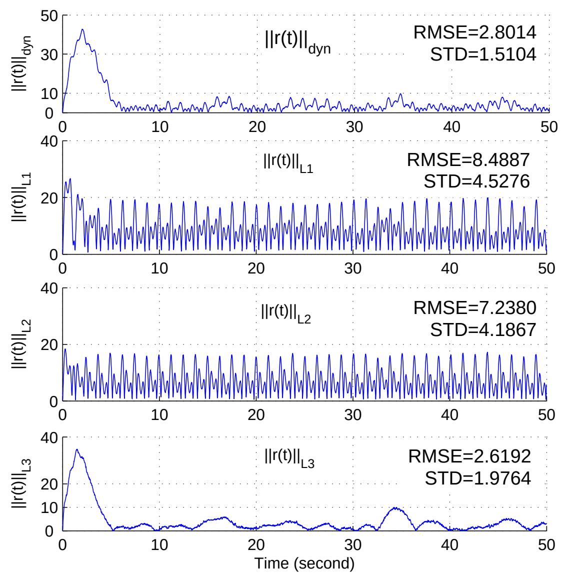

Fig. 5. Fault free residuals corresponding to the dynamic filter , the H_∞/H_∞ filter L_1, static filter L_2 and disturbance estimation approach L_3, respectively.

The disturbance attenuation performance is also evaluated in terms of the root mean square error (RMSE) and the standard deviation (STD):

$$\begin{aligned} RMSE = &\quad E\{\|y(t) - \hat{y}(t)\|\} = E\{\|r(t)\|\} \\ STD = &\quad \sqrt{E\{(\|r(t)\| - RMSE)^2\}} \end{aligned} \tag{41}$$

where $\|y\|$ denotes a vector norm $\sqrt{\sum_i |y_i|^2}$ and $E\{\}$ denotes the mathematical expectation. The RMSE/STD values are also shown in Fig. 5.

B. Residuals of abrupt actuator faults

In order to simulate the happenings of two successive abrupt actuator faults at two input channels respectively, the fault function is represented as $f_a(t) = [f_1(t)\ f_2(t)]^T$ and

$$f_1(t) = \begin{cases} 0 & (t < 20) \\ 0.05 & (t \geqslant 20) \end{cases} \quad f_2(t) = \begin{cases} 0 & (t < 30) \\ 0.05 & (t \geqslant 30) \end{cases}$$

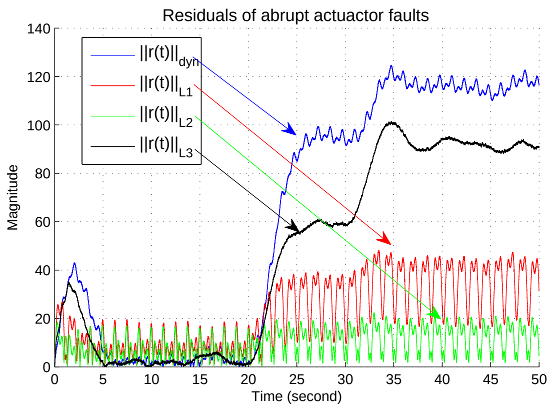

Fig. 6. Residuals of the dynamic filter, H_∞/H_∞ filter L_1 and static filter L_2 in the case of two abrupt actuator faults at 20s and 30s, respectively

The residuals of the dynamic filter, static filter L_1 and L_2, disturbance estimation are depicted in Fig. 6, where two step increases in $\|r(t)\|_{dyn}$ can be seen clearly within 1 second after each fault occurs. The filters L_1 and L_2, however, fail to detect such abrupt faults. Although the peak values of the residuals of these static filters present step increases, there is no clear interval between the normal residuals and faulty residuals. Therefore, missed alarms may exist.

C. Residuals of an incipient actuator faults

In this trial, the gradual fault injected to the input signal is $f_a(t) = [f_1(t)\ f_2(t)]^T$ and

$$f_1(t) = \begin{cases} 0 & (t < 20) \\ 0.0025(t-20) & (k \geqslant 20) \end{cases} \quad ; \quad f_2(t) = 0 \tag{42}$$

and the corresponding residuals are shown in Fig. 7.

To illustrate the fault detection performance, the plant outputs under such a fault are also plotted in Fig. 7. Due to the large output values and the small fault size, the changes in the outputs is difficult to

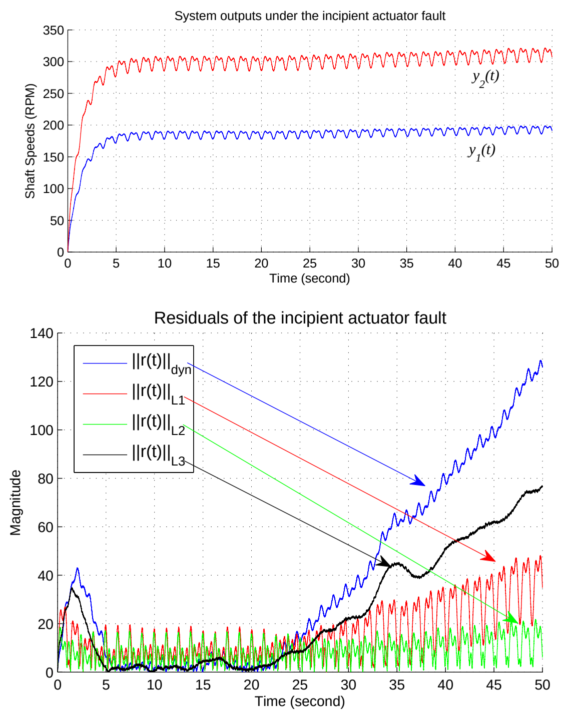

Fig. 7. Plant outputs and residuals under an incipient actuator fault at the first input channel. The actuator fault occurs at 20 second with slope rate 0.005

find. However, this fault can be detected clearly from the dynamic filter residuals. $\|r(t)\|_{dyn}$ responses the incipient fault (42) with a fairly straight line increasing at about 4 units per second. Whereas $\|r(t)\|_{L1}$ is with significant disturbances and its increase rate is only 0.7 unit per second. This comparison verifies that the dynamic filter is able to detect smaller gradual fault earlier and avoid missed alarm as much as possible.

A comparison between the proposed method and the classic disturbance estimation based method as shown in Figure 6-7, where the $||r(t)||_{L3}$ line represents the residual response of disturbance estimation method. As the magnitude of the residual of the proposed method (see $||r(t)||_{dyn}$) is larger than that of $||r(t)||_{L_3}$ after the fault occurs, it can be concluded that the proposed method is more sensitive to the fault and is therefore better in terms of fault detection performance.

VI. Conclusion

In this paper, a systematic study of the dynamic filter's zeros is presented. The properties of the proposed dynamic filter are analyzed and its capacities for fault detection are illustrated in simulations.

The possibility of zeros assignment in dynamic filter design are given and proved.

The proposed dynamic filter differs from the classical static filter (and Kalman Filter) whose gain matrix is a constant coefficient matrix. A dynamic gain is introduced into the dynamic filter design. and it is proved that a dynamic filter introduces additional zeros and shifts the system poles from $eig(A)$ to $eig(\tilde{A})$. Although this technique increases system order and complexity, it brings more freedom for filter design. In the proposed dynamic filter design approach, the additional degrees of design freedom are used to assign the additional zeros to desired places for attenuating disturbances further.

In the application, low frequency disturbances (5 rad/sec) need to be attenuated and the detection of both the actuator and sensor faults is required. Simulation results have shown that the proposed zeros assignment method comes with reasonably good disturbance attenuation. Hence, we are able, on one side, to formulate and get a better filter in the sense of robustness and disturbance rejection, and, on the other side, to obtain new insight into the filter construction for fault detection.

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